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# A THEORY FOR HEAT TRANSFER TO ANNULAR TWO-PHASE, TWO-COMPONENT FLOW

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**Abstract**—A theory based on an axisymmetric model with no liquid entrainment was developed to allow predictions of local heat-transfer coefficients. The theory treated the heat and mass transfer at the wave-roughened gas–liquid interface by using theoretical results from heat transfer to rough tubes. In comparing the heat transfer predictions with the measured coefficients, 85 per cent of the predictions were within ± 60 per cent of recently measured values from horizontal flow. The best agreement was observed in the flow regime for which the liquid film is expected to be most nearly symmetric as was assumed in the analytical model.

## NOMENCLATURE

$c_p$, specific heat [Btu/lb°F];
$D$, tube diameter [ft];
$f$, friction factor defined by $\Delta P = (fL\rho\bar{u}^2/D2)$;
$f_0$, friction factor for a smooth tube;
$h$, local heat transfer coefficient [Btu/hft$^2$ °F];
$h'$, local mass transfer coefficient [ft/h];
$h_e$, specific enthalpy [Btu/lb];
$h_{fg}$, constant pressure heat of vaporization [Btu/lb];
$\dot{m}$, mass flux (lb/hft$^2$), usually refers to evaporation process;
$Nu$, Nusselt number;
$Nu'$, Nusselt number for mass transfer;
$P$, pressure [lb/ft$^2$];
$p$, partial pressure [lb/ft$^2$];
$Pr$, Prandtl number;
$\dot{q}$, heat flux [Btu/hft$^2$];
$Re$, Reynolds number;
$T$, temperature;
$u$, axial velocity [ft/s];
$\dot{w}_a$, air flow rate [lb/s];
$\dot{w}_L$, water flow rate [lb/s];
$X$, Lockhart–Martinelli two-phase parameter. Square root of the ratio of pressure drop for liquid flowing alone to gas flowing alone in a pipe of specified diameter for the specified phase flow rates;
$y$, distance from tube wall [ft];
$\Phi_g$, $\Phi_l$, Lockhart–Martinelli parameters equal to square root of ratio two-phase pressure drop to single-phase pressure drop for same tube diameter;
$\rho$, density [lb/ft$^3$];
$\mu$, absolute viscosity [lb$_m$/ft s]
$\nu$, kinematic viscosity equal to $\mu/\rho$;
$\lambda$, molecular diffusivity for species [ft$^2$/h].

Subscripts

0, wall value;
$bf$, bulk liquid or film value;
$i$, value at liquid–gas interface;
$b$, bulk gas value;

$F$, final equilibrium or mixing cup state;
$g$, gas phase;
$l$, liquid phase.

## INTRODUCTION

In the field of two-phase flow most data useful for engineering purposes has appeared in the form of correlations of prior experimental results. Predictions of pressure drop and heat-transfer coefficients from these correlations are subject to great uncertainties, $\pm 60$ per cent, and the correlations are usually valid only within a limited flow range. Because of the complexities of two-phase flows, it is unlikely that any theories based entirely on first principles will in the near future yield results which are in truly good agreement with experiment. However, analytical efforts in the area can be expected to lead to better understanding of these flows and will hopefully suggest refinements, generalizations, or extensions of presently accepted correlations.

To date, no satisfactory mechanism-oriented analytical model or theory has been developed for heat transfer to annular two-phase, two-component flow. Several investigators [1–4] have developed models for heat transfer in two-phase flows, but none of these contain the elements necessary to properly treat horizontal annular two-phase, two-component flows.

In view of the need for improved understanding and better predictive methods, the present work is directed toward development of an analytic model which allows prediction of heat transfer rates from a knowledge of flow parameters and pressure drops. The model is restricted to annular two-phase, two-component flows since present knowledge indicates that transfer processes are dependent upon flow pattern.

## ANALYSIS

*General*

The system of interest is shown schematically in Fig. 1. The local heat-transfer coefficient under study is defined as

$$h = \dot{q}/(T_0 - T_F). \tag{1}$$

In this definition $T_F$ is an equilibrium temperature, i.e. mixing cup, with phase change considered for the flow at the appropriate axial location. This type of equilibrium temperature is the most easily measured fluid temperature. With this definition the problem reduces to development of adequate models to approximate velocity and temperature distributions, and mass and heat transfer processes in sufficient detail to allow computation of a $T_F$ value. These distributions are dependent upon flow rates and properties. The models developed assumed axial symmetry and no entrainment of liquid by the gas. While properties of water and air are used to allow comparison with the data of [5], this combination is considered representative of a broad class of substances.

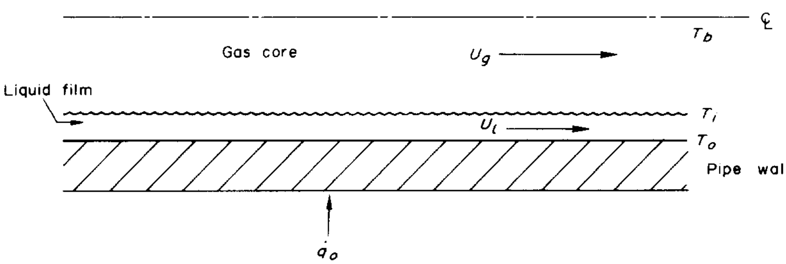


Fig. 1. Diagram of model for system.

*The model for liquid film*

To start the analysis, it is assumed that $\dot{q}_0$, $T_0$, $\dot{w}_L$, $\dot{w}_a$ are known. Further, the wall shear $\tau_0$, can be approximated either from pressure drop measurements or by the use of one of the improved correlations or theories, which are discussed in detail in [6].

If it is assumed that the velocity distribution in the symmetric liquid film is that which would exist in an annular portion of a full tube of liquid flowing with identical wall shear, then the turbulent velocity profile can be integrated until the flow rate $\dot{w}_L$ is consumed.

The profile can be integrated in a generalized way by using the dimensionless variables defined by $u^+ = u/u^*$, $y^+ = (\rho u^* y/\mu)$, where $u^* = (\tau_0/\rho)^{\frac{1}{2}}$, the friction velocity. For a film that is thin with respect to tube radius, we get

$$W^+(y^+) = \frac{\dot{w}_L}{\pi D \mu} = \int_0^{y^+ i} u^+(t)\,\mathrm{d}t\,. \quad (2)$$

Then $W^+$ is a dimensionless flow rate parameter, and depends only on $y_i^+$, the dimensionless film thickness, since the universal turbulent velocity profile, $u^+(y^+)$, is a known function as given, for example, by Van Driest [7] in a comparatively simple form:

$$u^+(y^+) =$$

$$\int_0^{y+} \frac{2\mathrm{d}t}{1 + \sqrt{\{1 + 4K^2t^2[1 - \exp(-t/A)]^2\}}}\,. \quad (3)$$

where $K = 0{\cdot}4$ and $A = 26$. Thus, the film thickness can be determined from the assumed values of $\dot{w}_L$ and $\tau_0$ by use of the above flow model.

With film thickness determined as indicated above, it is possible to use a universal temperature profile for turbulent flow to approximate the temperature of the liquid–gas interface. By assuming that eddy diffusivities for momentum and heat are equal in the liquid films, and using van Driest's expression for eddy diffusivity [7], the result is the following expression for the nondimensional temperature, $T^+$,

$$T^+ = \frac{(T_0 - T)\,c_p \rho u^*}{\dot{q}_0}, \quad (4)$$

where the properties and $\dot{q}_0$ are evaluated at the wall. Letting

$$B(y^+) = \sqrt{\{1 + 4K^2t^2[1 - \exp(-y^+/A]^2\}}$$

and with $K = 0{\cdot}4$ and $A = 26$ as before, we get

$$T^+(y^+, Pr) = \int_0^{y^+} \frac{\mathrm{d}s}{(1/Pr) + \frac{1}{2}[B(s) - 1]}\,. \quad (5)$$

By using equation (5), either through specific numerical integration for the given $Pr$ and dimensionless film thickness $y_i^+$, or by using curves prepared in advance, $T^+$ at the interface, $T_i^+$, can be found and then $T_i$ since $T_0$, $c_p$, $\rho$, $u^*$ and $\dot{q}_0$ are all known.

*Transfer processes from the interface to the gas core*

In an annular two-phase flow, the pressure drop and wall friction are greater than for either phase flowing singly. This increased wall friction is usually attributed to increased roughness felt by the gas because of surface waves on the liquid. By analogy with single-phase flow, it can be expected that the increased roughness would effect the heat and mass transfer.

The effect of wall roughness elements upon heat transfer in single phase flow has been studied by Nunner [8] and Kolar [9]. Both of these workers found that the Nusselt number for rough wall heat transfer could be expressed as a function of the flow rate, fluid properties, and the actual pressure drop. These works suggest that the Nusselt number is not strongly dependent upon the form of the roughness elements. Additionally, the usual Reynolds analogy does not hold because of wall friction being dependent on form drag. While the works of Nunner and Kolar are in good agreement.

Nunner's results, a generalization of the Prandtl–Taylor relation, are used:

$$Nu = (f/8)\,Re\,Pr/\left[1 + \left(\frac{u_l}{\bar{u}}\right)\left(\frac{fPr}{f_0} - 1\right)\right]$$

where $u_l/\bar{u}$ is the ratio of the velocity at the edge of the laminar sublayer to the mean gas velocity. If the empirical relation of Hofmann [10], $u_l/\bar{u} = 1{\cdot}5\ Re^{-\frac{1}{8}}\ Pr^{-\frac{1}{6}}$, is used, the following equation results:

$$Nu = (f\,Re\,Pr/8)/\left[1 + 1{\cdot}5\,Re^{-\frac{1}{8}}\,Pr^{-\frac{1}{6}}\left(\frac{Prf}{f_0} - 1\right)\right] \qquad (6)$$

which permits approximation of the interface heat transfer coefficient.

Mass transfer at the interface can be expressed as $\dot{m} = h'(\rho_i - \rho_b)$ where the $\rho$'s indicate specie concentration. Use of the perfect gas law yields $\dot{m} = (h'/RT_m)\,(p_i - p_b)$ where $T_m = (T_i + T_b)/2$. If, as is usual, the transfer of heat and mass are considered analogous, then replacement of the Prandtl number by the Schmidt number $Sc$ in equation (6) will yield the mass transfer Nusselt number $Nu'$, defined as $Nu' = h'D/\lambda$. If the interface mass flux is small, then for small values of the ratio of partial to total pressures, the following interface relation is valid:

$$\dot{q} = h_i(T_i - T_b) + \frac{h_i' h_{jg}}{RT_m}(p_i - p_b)\,. \qquad (7)$$

The vapor pressure at the interface is assumed to be the saturation pressure corresponding to the interface temperature. Hence, values of $p_i$ and $h_{jg}$ can be established, and only $T_b$ and $p_b$ remain unknown. While other alternatives were explored, the assumption that the bulk gas was saturated with water vapor proved to be most satisfactory and allows equation (7) to be solved for $T_b$ by iteration. All predictions in this paper are based on the assumption of a saturated gas stream.

*Final equilibrium temperature*

The prior analysis provides information equivalent to that of knowing the velocity, temperature, and water vapor concentration distributions of the two phases in sufficient detail to determine the equilibrium or mixing cup temperature of the two-phase mixture.

The equilibrium condition may be expressed by the condition

$$\int_s \vec{V}.\vec{n}\left(uh_e + \frac{u^2}{2}\right)\mathrm{d}s = 0\,. \qquad (8)$$

The region with boundaries $s$ consists of an adiabatic section of conduit and a downstream flow of fluid out of the section is characterized by a single temperature $T_F$ and the gas present is saturated with water vapor.

Because the integrand above is zero at the solid wall and because the integral everywhere else can be evaluated by using average values already computed, the condition expressed in equation (8) can be used to solve for $T_F$. Accordingly,

$$T_F = \frac{(c_{pa} + W_1)\,T_b + [(\dot{w}_L/\dot{w}_a) + W_2 - W_1] \times T_{bj} + (W_1 - W_2)\,h_{jg}}{[c_{pa} + W_2 + (\dot{w}_L/\dot{w}_a)]} \qquad (9)$$

where the change in kinetic energy has been neglected and the specific heat of water has been set equal to one. $W_1$ and $W_2$ are the specific humidities of the gas stream at the upstream and mixing cup locations, respectively, and $\dot{w}_L$ is the flow rate of liquid at the mixing location. An average $h_{jg}$ is used for the appropriate temperature range. Equation (9) cannot be solved explicitly for $T_F$ because of the temperature dependence of $W_2$ on the right-hand side but it can be solved readily by plotting or iteration. The heat-transfer coefficient can then be computed according to the above model from the definition, $h = \dot{q}_0/(T_0 - T_F)$.

## COMPARISONS OF THE PREDICTIONS OF THE THEORY WITH EXPERIMENTAL DATA

Except for the results reported by the present authors in [5], relatively little published data exists for heat transfer to two-phase, two-component flow in the annular regime.

The experimental results of [5] were found to be well correlated by plotting $h/\phi_l$, a parameter suggested by Levy, against $X$. Although $h/\phi_l$ and $X$ do not appear as explicit parameters of the present analysis, it is nevertheless useful and of interest to compare the predictions and the experimental results of [5] in terms of these common two-phase flow parameters.

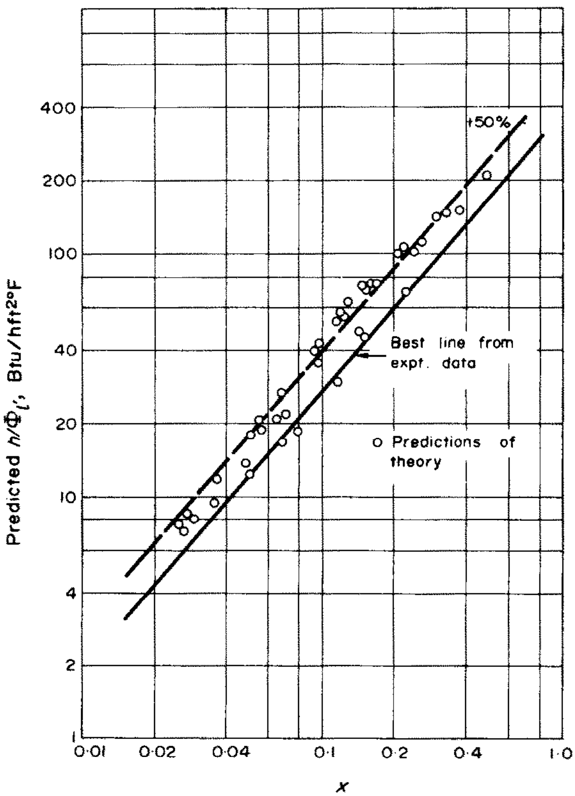


FIG. 2. Comparison of the predictions of the unmodified heat transfer theory with experimental results on the basis of $X$.

Figure 2 shows such a comparison where the solid line represents the best line through the experimental data of [5]. In [5], scatter in the experimental data, on the order of $\pm 20$ per cent was observed about this line. The predictions follow the same trend with $X$ as the data. The agreement between the theory and data, though not outstanding, is reasonable considering the simplifications and approximations made in the model and the overall complexity of the two-phase system. The best agreement is observed for the smaller values of $X$ corresponding to the regime where the liquid film is expected to be most nearly symmetric.

In [6] the predictions were corrected by empirically correlating the effects of film assymmetry and entrainment. The corrected predictions were within $\pm 25$ per cent of the experimental data of [5]. The details of these developments are too lengthy for the present note. Also in [6]. the theory was applied to conditions beyond the range of known experimental results to predict the effects of varying heat flux and tube temperature upon the heat-transfer coefficient.


## ACKNOWLEDGEMENTS

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## UNE THEORIE DE TRANSFERT THERMIQUE POUR UN ECOULEMENT ANNULAIRE BIPHASIQUE A DEUX COMPOSANTS

**Résumé**—Une théorie basée sur un modèle axisymétrique sans entrainement de liquide a été dévelopée pour déterminer le coefficient local de transfert thermique. Cette théorie considère le transfert thermique et massique à l'interface ondulé gaz–liquide en s'appuyant sur des résultats théoriques de transfert thermique pour des tubes rugueux. En comparant les estimations du transfert thermique aux coefficients mesurés, ces estimations sont dans la proportion de 85 pour cent intérieures à la région $\pm 60$ pour cent des valeurs récemment mesurées dans le cas d'un écoulement horizontal. Le meilleur accord est observé dans le régime d'écoulement pour lequel le film liquide est le plus proche de la symétrie supposée dans le modèle analytique.

## EINE THEORIE FÜR DEN WÄRMEÜBERGANG IM RINGSPALT BEI ZWEIPHASEN-, ZWEIKOMPONENTEN-STRÖMUNG

**Zusammenfassung**—Auf der Basis eines axialsymmetrischen Modells ohne Flüssigkeitseinspeisung wurde eine Theorie zur Bestimmung von lokalen Wärmeübergangszahlen entwickelt. Die Theorie behandelt den Wärme- und Stoffübergang an der welligen Gas-Flüssigkeits-Grenze unter Verwendung der theoretischen Ergebnisse des Wärmeübergangs an rauhe Rohrwände. Ein Vergleich der Theorie mit gemessenen Koeffizienten zeigte, dass 85% der berechneten Werte innerhalb $\pm$ 60% im Bereich der gemessenen bei horizontaler Strömung lagen. Die beste Übereinstimmung wurde in dem Strömungszustand beobachtet, für den der Flüssigkeitsfilm weitgehend symmetrisch ist, wie dies im analytischen Modell angenommen war.

## ТЕОРИЯ ТЕПЛООБМЕНА ДЛЯ ДВУХФАЗНОГО ДВУХКОМПОНЕНТНОГО КОЛЬЦЕВОГО ПОТОКА

**Аннотация**—На основе осесимметричной модели, не учитывающей начального участка, разработана теория, позволяющая рассчитать локальные коэффициенты теплообмена. Теория рассматривает теплои массообмен на волнообразной поверхности раздела газ-жидкость, используя теоретические результаты по теплообмену в шероховатых трубах. Сравнение расчетных коэффициентов теплообмена с измерениями показало, что 85% расчетных данных согласуются с последними измерениями в горизонтальном потоке с точностью до $\pm 60$%. Наилучшее совпадение наблюдалось в режиме течения, в котором пленка жидкости наиболее симметрична, что соответствует принятой модели.